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Modeling of surface tension of microdroplets

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This paper investigates the modeling of surface tension of micro droplets. A parallel volume of fluid code has been developed to simulate interfacial flows with surface tension and surfactant. The continuity, momentum and energy equations were used to formulate the governing equations for the problem. The governing equations were dimensionalised and the resulting equations were solved with variable separable and Eigen function expansion method. Pressure gradient was varied and also considered. The results obtained show that harmonic pressure gradient is smooth, more stable and shows a good agreement with the experimental data and theory.

Key words: Surface tension, pressure gradient, surfactant, micro droplets and Eigen function expansion.

INTRODUCTION

Interfacial flows with surface tension are encountered frequently in industrial and engineering applications, a prototypical example being material processing. Accurate modeling of such flows is challenging because of the discontinuity in material properties across the interface and because of interfacial boundary conditions due to surface tension forces (Ashgriz and Poo, 1990).

The role of interfaces in our daily life becomes increasingly apparent. The advancement achieved in surface chemistry has led to countless applications in a multitude of industries, for example, interfaces are crucially important in pharmaceutics, biotechnology and biomedicine (Jiang et al., 1992). Due to this increased interest, there is a growing need for specific interfacial consideration that can be used routinely to solve pharmaceutical problems and improve product quality (Anahita et al., 2009).

In order to meet challenges and develop new and better performing products, knowledge of surface tension is of utmost importance. Surface chemistry deals with chemical processes at the interface between two phases, therefore surface properties and their manipulation in certain application area play important roles (Hirt and Nichols, 1981). This work addresses the applications of surfactants to surface tension of micro droplets in Medicine and pharmaceutical industry (Sorjamaa et al., 2004).

The continuum surface force (CSF) method of Brackbill et al. (1992) has been employed extensively over the last 15 years to model surface tension in various fixed (Eulerian) mesh formulations for interfacial flows, in particular in the volume-of-fluid (VOF) since this present work is concentrating on Modeling of surface tension of Micro droplets (effect of surface tension with surfactants on living and nonliving). The separation of variables and Eigen function expansion (Analytical Method) method with combinations of some methods discussed above were used to determine the impact of surface tension on micro droplets (Attinger, 2000).

Surfactants are surface-active molecules that selectively adhere to interfaces. Surfactants typically consist of a hydrophilic head and a hydrophobic tail (detergents are common examples). Surfactants play a critical role in numerous important industrial and biomedical applications ranging from enhanced oil recovery (Yabe and Xiao, 1995) and pulmonary function (Prisle et al., 2008).

Surfactants are adverted and diffused along interfaces by the motion of the liquid and by molecular mechanisms respectively by (Sethian and Smereka, 2003). In this work, surface tension depends on the surfactant distribution and the pressure gradient of different form to determine the effects of surfactant with and without pressure gradient with minimum temperature through the
System (Li et al., 1998).

**MATHEMATICAL FORMULATION**

The general continuity equation is given by (Gu et al., 2004)

\[ \frac{\partial u}{\partial x} = 0 \]  (1)

In the present work we have considered the continuity equation in one-dimension, so that,

\[ \frac{\partial u}{\partial y} = 0, \]

where \( u \) is the velocity and \( y \) is the spatial direction.

The flow is governed by the incompressible Navier Stokes equations (Gu et al., 2006; Momentum and Energy Equations)

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + \mu \frac{\partial^2 u}{\partial y^2} \]  (2)

\[ \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + \mu \left( \frac{\partial u}{\partial y} \right)^2 \]  (3)

Equations 2 and 3 are Momentum and Energy equations respectively, where \( p, \rho \) and \( \mu \), and \( v \) are pressure, density, viscosity and velocity respectively.

\[ \mu \frac{\partial u}{\partial y} = \sigma_f \frac{\partial T}{\partial x} \]  (4)

Where, \( \sigma_f \) is the surface force with temperature.

In order to make the equation to be usable and errors free one can introduce the dimensionless variables as following:

Let,

\[ \theta = \frac{T}{T_0}, \quad T = \theta T_0 \]

\[ U = \frac{u}{u_0}, \quad u = U u_0 \]

\[ Y = \frac{y}{L}, \quad y = Y L \]

\[ T = \frac{t}{t_0}, \quad t = T t_0 \]

\[ X = \frac{x}{L}, \quad x = X L \]

Putting the dimensionless variables into Equation 2 yields:

\[ \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial Y} = G + \frac{1}{Re} \frac{\partial^2 U}{\partial Y^2} \]  (5)

Where, \( Re = \frac{u L}{\mu} \) (Reynolds number)

\[ G = -\frac{t_0}{u_0} \frac{1}{\rho} \frac{\partial P}{\partial x} \] (Pressure gradient)

\[ V_0 = V \frac{t_0}{L} \] (Suction parameter)

Putting the dimensionless variables into Equation 3 yields:

\[ \frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} + Ec \left( \frac{\partial U}{\partial Y} \right)^2 \]  (6)

where,

\[ Pr = \frac{L^2}{K t_0} \] (Prandtl number)

\[ Ec = \frac{\mu U^2 t_0}{L^2 T_0} \] (Eckert number)

Putting the dimensionless variables into Equation 4 yields:

\[ \frac{\partial U}{\partial Y} = \gamma \]  (7)

\[ \frac{\partial U}{\partial Y} = \sigma_f \epsilon \frac{\partial \theta}{\partial X} \]

where,

\[ \epsilon = \frac{\sigma_f T_0}{\mu U_0} \]  (8)

and

\[ \epsilon \left( \frac{\partial \theta}{\partial X} \right) = \gamma \]  (9)

where, \( G = 0, \epsilon, G_0 \cos \omega t \) and \( t, \theta, k \) and \( \gamma \) are time, temperature, thermal conductivity and surface tension respectively, \( \frac{\partial \theta}{\partial X} \) is constant.

Here are the Initial and Boundary conditions:

\[ \frac{\partial U}{\partial Y}(0, t) = \gamma \quad 0 < Y < 1 \]

\[ U(L, t) = 0, \quad \theta(L, t) = 0 \]  (10)

\[ U(0, 0) = 0, \quad \theta(0, 0) = 0 \]

\[ Y = L = 1 \]

**METHOD OF SOLUTIONS**

The following equations are the dimensionized partial
differential equations coupled systems of momentum and energy equations of one dimension from Equations 5 and 6 respectively:

\[
\frac{\partial U}{\partial t} + v_0 \frac{\partial U}{\partial y} = G + \frac{1}{Re} \frac{\partial^2 U}{\partial y^2}
\]

Effect of pressure gradient I

In this situation pressure gradient is taking to be zero (\(G = 0\)). Therefore, Equation 5 becomes

\[
\frac{\partial U}{\partial t} = \frac{1}{Re} \frac{\partial^2 U}{\partial y^2} - v_0 \frac{\partial U}{\partial y}
\]

(11)

So, \(u, t, v_0\) and \(Re\) are velocity, time suction parameter and Reynolds number respectively.

\[
\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + Ec \left( \frac{\partial U}{\partial y} \right)^2
\]

Equation 6 is an energy, one dimension and nonlinear equation where \(\theta\) is the temperature, \(Pr\) is the Prandtl number and \(Ec\) is the Eckert number. The following are the initial and boundary conditions for the velocity and temperature.

\(U'(0, t) = \gamma ; \quad U(L, t) = 0 ; \quad U(y, 0) = 0 \quad 0 < y < L\)
\[\theta(0, t) = 0 ; \quad \theta(L, t) = 0 \quad (t > 0)\]

METHOD OF VARIABLE SEPARABLE AND EIGENFUNCTION EXPANSION

The above named methods were used to determine the velocity as the temperature maintained zero. Let, \(U(y, t) = u_a(y) + v(y, t)\). From Equation 11,

\[
\frac{\partial U}{\partial t} = \frac{1}{Re} \frac{\partial^2 U}{\partial y^2} - v_0 \frac{\partial U}{\partial y}
\]

\(U'(0, t) = \gamma ; \quad U(L, t) = 0 ; \quad U(y, 0) = 0\)

\[
\frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u_a(y)}{\partial y^2} + \frac{\partial^2 v(y, t)}{\partial y^2}
\]

(12)

\[
\frac{\partial^2 u}{\partial y^2} = V_0 \frac{\partial u_a}{\partial y} + V_0 \frac{\partial v}{\partial y}
\]

(13)

Substituting Equations 12 and 13 into Equation 11

\[
\frac{\partial v}{\partial t} = \frac{1}{Re} \frac{\partial^2 u_a}{\partial y^2} + \frac{1}{Re} \frac{\partial^2 v}{\partial y^2} - V_0 \frac{\partial u_a}{\partial y} - V_0 \frac{\partial v}{\partial y}
\]

(14)

make some arrangements

\[
\frac{1}{Re} \frac{\partial^2 u_a}{\partial y^2} - V_0 \frac{\partial u_a}{\partial y} = 0
\]

(15)

\[
\frac{\partial v}{\partial t} = \frac{1}{Re} \frac{\partial^2 v}{\partial y^2} - V_0 \frac{\partial u}{\partial y}
\]

(16)

Multiply Equation 16 by \(Re\) then,

\[
\frac{\partial^2 u_a}{\partial y^2} - V_0 \frac{\partial u_a}{\partial y} = 0
\]

(17)

\[
\frac{\partial^2 u_a}{\partial y^2} - a \frac{\partial u_a}{\partial y} = 0
\]

(18)

Where, \(a = V_0 \frac{\partial u_a}{\partial y} = m\) to have a quadratic equation

\[m^2 - ma = 0\]

\[m = 0 \quad \text{or} \quad a\]

The general solution of Equation 18 becomes:

\[u(y) = A + B e^{ay}\]

(19)

Differentiating equation (19) and applying the boundary condition becomes

\[U(y) = \frac{Z}{a} \left[ e^{ay} - \ell^{al} \right]\]

(20)

Let, \(V(y, t) = X(y) T(t)\). From Equation 16:

\[
\frac{\partial v}{\partial t} = \frac{1}{Re} \frac{\partial^2 v}{\partial y^2} - V_0 \frac{\partial u}{\partial y}
\]

\[
\frac{T'(t)}{T(t)} = -\lambda^2
\]

\[
T'(t) + \lambda^2 T(t) = 0
\]

(21)

\[
\frac{1}{Re} \frac{X'(y)}{X(y)} - V_0 \frac{X'(y)}{X(y)} = -\lambda^2
\]

\[
\frac{1}{Re} m^2 - V_0 m + \lambda^2 = 0
\]

(22)
Let,

\[ m = \frac{a \pm i \sqrt{4 \text{Re} \lambda^2 - a^2}}{2} \]

Let, \( \alpha = \frac{\sqrt{4 \text{Re} \lambda^2 - a^2}}{2} \)

The general equation becomes:

\[ v(y) = e^{\pm i \alpha y} \left[ A \sin \alpha y + B \cos \alpha y \right] \] (23)

\[ v'(y) = e^{\pm i \alpha y} \left[ A \sin \alpha y + B \cos \alpha y \right] + \lambda e^{\pm i \alpha y} \alpha \cos \alpha y - B \alpha \sin \alpha y \] (24)

\[ v(0, t) = 0 \] (25)

\[ B = -\frac{2A \alpha}{a} \]

\[ v(L, t) = 0 \]

\[ v(L) = e^{\pm i \alpha L} \left[ A \sin \alpha L + B \cos \alpha L \right] = 0 \] (26)

\[ A \sin \alpha L + B \cos \alpha L = 0 \] (27)

Substitute Equations 25 into 27:

\[ A \left[ \sin \alpha L + \frac{2a}{a} \cos \alpha L \right] = 0 \] (28)

For non-trivial solution, \( A \neq 0 \) therefore Equation 28 becomes:

\[ \sin \alpha L - \frac{2a}{a} \cos \alpha L = 0 \] (29)

\[ \tan \alpha L - \frac{2a}{a} = 0 \]

Let \( z = \alpha L \)

\[ \therefore \tan z = \frac{2a}{a} \] (30)

Equation 22 becomes:

\[ v(y) = e^{\pm i \alpha y} \left[ A \sin \frac{Z_n y}{L} - \frac{2AZ_n}{aL} \cos \frac{Z_n y}{L} \right] \] (31)

\[ v(y, t) = \sum A_n e^{\pm i \beta_n \frac{y^2}{L}} \left[ \sin \frac{Z_n y}{L} - \frac{2Z_n}{aL} \cos \frac{Z_n y}{L} \right] \] (32)

\( Z_n \) is the Eigen value

Since, \( U(y, t) = u_\alpha(y) + v(y, t) \)

Therefore the velocity solution with the following parameter becomes:

\[ u(y, t) = -0.03595417 \times 1.4881 \times \sin(z_1 y - 8z_1 \cos z_1 y) + \frac{Z}{a} \left[ e^{\alpha L} - e^{\alpha L} \right]. \] (33)

The value of \( z_1 \) and \( z_2 \) was considered because they are real values.

So, \( z_1 \) and \( z_2 \) values are 1.4881 and -1.4881 respectively.

Since \( a = \text{Re} V_0 \) where \( \text{Re} = 1, L = 1 \) and \( V_0 = 1.6166 \)

Since \( \alpha = \frac{\sqrt{4 \text{Re} \lambda^2 - a^2}}{2} \)

\[ \lambda = \sqrt{\frac{4 \alpha^2 + a^2}{4 \text{Re}}} \]

Different values of \( V_0 \) (suction parameter) that determined \( \alpha \) were computed to generate different values of \( \lambda \) (surfactant).

Therefore:

\[ \lambda = 1.4933, 1.5090, 1.5346, 1.5699, 1.6140, 1.6664 \]

Computing all the following parameters:

\[ z_n = 1.4881, \lambda = 1.4933, a = 0.25, L = 1, t = 0.5, n = 1 \]

\[ A_n = -0.035954129757 \]

Therefore, Equation 33 becomes:

\[ u(y, t) \]

As Equation 33 remains the general solution of the momentum equation. From Equation 6:

\[ \frac{\partial \theta}{\partial t} = \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial y^2} + Ec \left( \frac{\partial U}{\partial y} \right)^2 \]

We considered the energy equation in this form

\[ \frac{\partial \theta}{\partial t} = \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial y^2} + q(y, t) \]

So, let \( q(y, t) = Ec \left( \frac{\partial U}{\partial y} \right)^2 \)

The general solution of momentum equation was partially differentiated with respect to \( y \) and square to yield:

\[ \left( \frac{\partial u}{\partial y} \right)^2 = A_0 \left( \frac{2z_0^2}{aL^2} \sin\left( \frac{z_0 y}{L} \right) - \frac{2z_0^2}{aL^2} \sin\left( \frac{z_0 y}{L} \right) \right)^2 \] (34)
The remaining part of the Equation 6 was solved with variable separable and Eigen function expansion

\[ \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \]

The general solution of energy equation is of the form:

\[ \theta(y,t) = \theta_1(r) X_1(y) + \theta_2(r) X_2(y) \]

So, the general solution of energy equation is:

\[ \theta(y,t) = \frac{\phi}{\pi^2} \left[ \int_0^L (1 - e^{-k^2y}) \sin(ky) dy + \frac{1}{4} \left( \int_0^L (1 - e^{-k^2y}) \sin(2ky) dy \right) \right] \]

(35)

\[ \phi = 2Ec \int_0^L \left[ A^2 e^{pr^2} \left( \sin \left( \frac{2ky}{L} \right) - \frac{2s}{aL} \cos \left( \frac{2ky}{L} \right) \right) \left( \sin \left( \frac{4ky}{L} \right) \right) \right] dy \]

\[ \theta = 0.01292535086 \gamma^2 Ec \]

When, \( n = 1, L = 1, a = 0.25, t = 0.5, \lambda = 1.4881 \) and \( k = 1 \)

Computing all the parameters the general solution is reduced to:

\[ \theta(y,t) = 0.0000217737 \sin(\pi y) + 0.0000350908(2\pi y) \]

The idea of some parameters came from the reviewed work on surface tension and surfactant. Although the values of the surfactant \( (\lambda) \) depend on the curvature and suction parameter, it was shown that the computational results of \( u(y,t) \) and \( \theta(y,t) \) in terms of surface tension \( \gamma \) depend on \( \lambda \). \( Ec = 0.6, t > 0 \quad 0 \leq y \leq 1 \).

**Effect of pressure gradient II**

Considering Equation 1:

\[ \frac{\partial u}{\partial t} = \frac{1}{Re} \frac{\partial^2 u}{\partial y^2} - \frac{V_0}{Re} \frac{\partial u}{\partial y} + G \]

When, \( G = \frac{\phi}{\pi^2} \int_0^L (1 - e^{-k^2y}) \sin(ky) dy + \frac{1}{4} \left( \int_0^L (1 - e^{-k^2y}) \sin(2ky) dy \right) \]

Here are the initial and boundary conditions:

\[ \frac{\partial u}{\partial y}(0,t) = \gamma; \quad U(L,t) = 0; \quad U(y,0) = 0 \]

Let, \( U = V(y) + W(y,t) \)

Therefore Equation 36 yields:

\[ \frac{\partial w}{\partial t} = \frac{1}{Re} \left[ \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right] - \frac{V_0}{Re} \left[ \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} \right] + G \cos \omega t \]

(37)

Equation 37 with the above boundary conditions yields:

\[ w(y,t) = \sum A_n \frac{\partial^2}{\partial z^2} \left( \sin \left( \frac{2n\psi y}{L} \right) \right) \]

(38)

So,

\[ A_n = \frac{\int_0^L (1 - e^{-k^2y}) \sin(ky) dy}{\int_0^L (1 - e^{-k^2y}) \sin(2ky) dy} \]

When, \( n = 1, \lambda = 1.4933, t = 0.5, Ec = 0.6 \)

Therefore,

\[ A_n = 0.03595413118 \gamma + 0.0370406307 \]

And the velocity reduces to:

\[ u(y,t) = 0.03595413118 \gamma + 0.0370406307 \]

When, \( G = \frac{\phi}{\pi^2} \int_0^L (1 - e^{-k^2y}) \sin(ky) dy + \frac{1}{4} \left( \int_0^L (1 - e^{-k^2y}) \sin(2ky) dy \right) \]

\[ \theta = 0.01292535086 \gamma^2 Ec \]

So, the Temperature becomes:

\[ \theta(y,t) = 0.0000217737 \sin(\pi y) + 0.0000350908(2\pi y) \]

**Effect of pressure gradient III**

When \( G = G_0 \cos \omega t \)

Where \( G_0 \) is a constant, \( \omega \) is an angular velocity and \( t \) is the time taken.

\[ \frac{\partial u}{\partial y}(0,t) = \gamma; \quad U(L,t) = 0; \quad U(y,0) = 0 \]

Let, \( U = V(y) + W(y,t) \)

Therefore Equation 36 yields:

\[ \frac{\partial w}{\partial t} = \frac{1}{Re} \left[ \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right] - \frac{V_0}{Re} \left[ \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} \right] + G_0 \cos \omega t \]

(39)

Here are the initial and boundary conditions:

\[ \frac{\partial u}{\partial y}(0,t) = \gamma; \quad U(L,t) = 0; \quad U(y,0) = 0 \]

Let, \( U = V(y) + W(y,t) \)

Therefore Equation 36 yields:

\[ \frac{\partial w}{\partial t} = \frac{1}{Re} \left[ \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right] - \frac{V_0}{Re} \left[ \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} \right] + G_0 \cos \omega t \]

(40)

Therefore the solution of Equation 40 becomes:

\[ w(y,t) = \sum A_n \frac{\partial^2}{\partial z^2} \left( \sin \left( \frac{2n\psi y}{L} \right) \right) \]

(41)

Let, \( \xi = \frac{4G_0 \cos \omega t}{2n\pi + a} \)

\[ w(y,t) = \sum A_n \frac{\partial^2}{\partial z^2} \left( \sin \left( \frac{2n\psi y}{L} \right) \right) \]

(38)
When, \( n = 1, a = ReV_0 = 0.25, t = 0.5, G_0 = 1, L = 1, \omega = 60^\circ \)
Therefore equation 42 yields:

\[
\frac{d\theta}{dt} + n^2 \pi^2 t = \zeta
\]  

(47)

So, the general solution for energy equation is given as:

\[
\theta(y, t) = \frac{\zeta}{n^2}[1 - \epsilon^{-n^2t}] \sin(ny) + \frac{\zeta}{n^2}[1 - \epsilon^{-n^2t}] \sin(2ny)  
\]

(48)

Therefore, Equation 48 becomes:

\[
\theta(y, t) = 0.09366458090(3.75\pi/138) + 0.03993223312 + 0.4279301022 \sin(3.143y) 
\]

RESULTS

The flow of an incompressible inviscid fluid with surface tension in the form of micro-droplet at the interface in a region bounded in a pipe has been studied. The Velocity field and Temperature distributions have been obtained and shown in figures for various values of parameters as the pressure gradient \( G \) is given as \(( G = 0, \theta' \text{and} G_0 \cos \omega \)). The following observations have been made.

Figures 1-4 show the velocity (U) and Temperature (\( \theta \)) profiles for various values of spatial direction (\( y \)) when the surface tension was fixed with different combinations values of surfactants \( \lambda \) and varied with different values of suction parameters \( V_0 \) as the pressure gradient \( G \) varies as \( G = 0, \theta' \text{and} G_0 \cos \omega \).

Figure 1a present the impact of different combinations of surfactant values (\( \lambda \)) when the pressure gradient is zero \(( G = \zeta \)) at the interface of micro-droplets with a value of surface tension \( \gamma \) 0.129. It is clear that the velocity profile (U) of the droplets is not stable when the spatial direction (\( y \)) of the system remain zero (0) but stable as \( y \) increases from 0.20 to 1.00 and velocity decreases from 0.2630928582 to 0.005795628681 with consistent increased in surfactant \( \lambda \). We also observed that different droplets with different velocity intercept as \( y \) increases therefore, the velocity of all droplets are close because of the surfactants so these really satisfied the stability of the system being the major challenge in droplets size. Figure 1b present the impact of different combinations values of suction parameters \( V_0 \) when the pressure gradient is zero \(( G = \zeta \)) at the interface of micro-droplets with each value of surfactant \( \lambda \) 1.4933 and surface tension \( \gamma \) 0.129. It is clear that the velocity profile (U) of the droplets is stable when the spatial direction (\( y \)) of the system remain zero (0) and also stable as \( y \) increases from 0.00 to 1.00 and velocity decreases from 0.03621297687 to 0.0005664892911 with consistent increase in suction parameters \( V_0 \). We also observed that different droplets with different velocity intercept as
Figure 1. Velocity profile for various values of (a) surfactants parameter as $G=0$ and $\gamma=0.129$; (b) suction parameter as $G=0$ and one surfactant value ($\lambda$) = 1.4933; (c) surfactants parameter as $G=\gamma$ and $\gamma=0.129$; (d) surfactants parameter as $G=G_0 \cos wt$ and $\gamma=0.129$; (e) suction parameter as $G=G_0 \cos wt$ and one surface tension ($\gamma$) = 0.129.

$y$ increases therefore, the velocity of all droplets are not close compare with Figure 1a. Figure 1c also present the impact of velocity profile for various values of surfactant ($\lambda$) when the pressure gradient is visible and also high (i.e., $G = \gamma$) at the interface of micro droplets with a value of surface tension ($\gamma$) = 0.129. It is clear that exponential $y$ (i.e., $\gamma$) is high in terms of value, so it repel the velocity of each micro droplet so, this also put the flow of micro droplets in the negative direction as the surfactant values ($\lambda$) increases. It is clear that the velocity profile ($U$) of the droplets is not stable when the spatial direction ($y$) of the system is zero (0) but stable as $y$ increases from 0.20 to 1.00 and velocity also increases from -1.816563551 to 0.003218384 with consistent increase in surfactant values ($\lambda$). Figure 1d also present the velocity profile for combinations of surfactant ($\lambda$) when the pressure gradient $G=G_0 \cos wt$ at the interface of micro-droplets with a value of surface tension ($\gamma$) 0.129. It is clear that the velocity profile ($U$) of the droplets is stable when the spatial direction ($y$) of the system increases from 0.00 to 1.00 and velocity decreases from 4.661315536 to 0.07285547853 with consistent increase in surfactant $\lambda$ which shows that Harmonic values as pressure gradients stabilized this system more than Constant and Exponential values. Figure 1e also present the velocity profile for combinations of suction parameter $V_0$ when the pressure gradient $G=G_0 \cos wt$ at the interface of micro-droplets with a value of surface tension ($\gamma$) 0.129. It is clear that the velocity profile ($U$) of the droplets is stable when the spatial direction ($y$) of the system increases from 0.00 to 1.00 as velocity also increases from -0.293114230 to -0.00336382891 with consistent increase in suction parameter $V_0$ which shows that Harmonic values as pressure gradients stabilized this system more than Constant and Exponential values though all the velocity values are negative but it is more consistent on this system.

Figure 2a present the impact of different combinations of surfactant values ($\lambda$) when the pressure gradient is zero ($G=0$) at the interface of micro-droplets with a value
of surface tension ($\gamma$) 0.139. It is clear that the velocity profile ($U$) of the droplets is not stable when the spatial direction ($y$) of the system remain zero (0) but stable as $y$ increases from 0.20 to 1.00 and velocity decreases from 0.2834876536 to 0.006244902226 with consistent increase in surfactant $\lambda$. Figure 2b present the impact of different combinations values of suction parameters $V_0$ when the pressure gradient is zero ($G=0$) at the interface of micro-droplets with one value of surfactant ($\lambda$) 1.5090 and surface tension ($\gamma$) 0.139. It is clear that the velocity profile ($U$) of the droplets is stable when the spatial direction ($y$) of the system remain zero (0) and also stable as $y$ increases from 0.00 to 1.00 and velocity decreases from 0.035369485 to 0.0005532943291 with consistent increase in suction parameters $V_0$. Figure 2c also present the impact of velocity profile for various values of surfactant ($\lambda$) when the pressure gradient is visible and also high ($\text{i.e. } G=G_0 \cos wt$) at the interface of micro droplets with a value of surface tension ($\gamma$) = 0.139. It is clear that exponential $y$ ($\text{i.e. } e^y$) is high in terms of value, so it repel the velocity of each micro droplet so, this also put the flow of micro droplets in the negative direction as the surfactant values ($\lambda$) increases. It is clear that the velocity profile ($U$) of the droplets is not stable when the spatial direction ($y$) of the system is zero (0) but stable as $y$ increases from 0.20 to 1.00 and velocity also increases from -1.836570753 to -0.04029801874 with consistent increase in surfactant values ($\lambda$). Figure 2d also present the velocity profile for combinations of surfactant ($\lambda$) when the pressure gradient $G=G_0 \cos wt$ at the interface of micro-droplets with a value of surface tension ($\gamma$) 0.139. It is clear that the velocity profile ($U$) of the droplets is stable when the spatial direction ($y$) of the system increases from 0.00 to 1.00 and velocity decreases from 4.627149182 to 0.07231988653 with consistent increase in surfactant $\lambda$. Figure 2e also present the velocity profile for combinations of suction parameter $V_0$ when the pressure gradient $G=G_0 \cos wt$ at the interface of micro-droplets with a value of surface tension ($\gamma$) 0.139. It is clear that the velocity profile ($U$) of the droplets is stable when the spatial direction ($y$) of the system increases

![Figure 2. Velocity profile for various values of (a) surfactants parameter as $G=0$ and $\gamma=0.139$; (b) suction parameter as $G=0$ and one surfactant value ($\lambda$)=1.5090; (c) surfactants parameter as $G=G_0 \cos wt$ and $\gamma=0.139$; (d) surfactants parameter as $G=G_0 \cos wt$ and one surface tension ($\gamma$)=0.139.](image)
Figure 3. Temperature profile for various values of (a) surfactants parameter as $Ec=0.6$, $G=0$ and $\gamma=0.129$; (b) suction parameter as $G=0$ and one surfactant value ($\lambda$)=1.4933; (c) surfactants parameter as $Ec=0.6$, $G=e^y$ and $\gamma=0.129$; (d) surfactants parameter as $Ec=0.6$, $G=G_0 \cos wt$ and $\gamma=0.129$; (e) suction parameter as $G=G_0 \cos wt$ and one surface tension ($\gamma$)=0.129.

from 0.00 to 1.00 as velocity also increases from -0.315836264 to -0.00362031391 with consistent increase in suction parameter $V_0$.

Figure 3a present the Temperature distributions of various values of surfactant ($\lambda$) with a value of surface tension ($\gamma$) = 0.129, when the pressure gradient is zero ($G=0$) at the interface of micro-droplets. It is clear that the Temperature profile ($\theta$) of the droplets maintain its peak when the spatial direction ($y$) of the system is 0.40 with maximum and minimum temperatures of 0.00003735355838 and 0.00002161708751 respectively, the reduction in temperature was due to consistent increase in surfactant $\lambda$ at the interface. We also observed that surfactant penetrate the interface with low temperature, the temperature of all droplets are close because of the surfactants. Figure 3b present the Temperature distributions of various values of suction parameter with a value of surface tension ($\gamma$) 0.129 and a surfactant ($\lambda$), when the pressure gradient is zero ($G=0$) at the interface of micro-droplets. This shows that the Temperature profile ($\theta$) of the droplets maintain its peak when the spatial direction ($y$) of the system is 0.40 with minimum and maximum temperatures of 0.00002277123153 and 0.0001344450715 respectively, the increase in temperature was due to less value of surfactant which will also increase the time of interfacial penetration. The temperatures of all droplets are not close. Figure 3c also present the different combinations of surfactant values ($\lambda$) at the interface of microdroplets when the pressure gradient ($G_0 \cos wt$). It is clear seen that the Temperature profile ($\theta$) of the droplets maintain its peak when the spatial direction ($y$) of the system is 0.40 with maximum and minimum temperatures of 0.00181275430 and 0.001049069443 respectively, the reduction in temperature was due to consistent increase in surfactant $\lambda$ at the interface but not as low compare to
Figure 4. Temperature profile for various values of (a) surfactants parameter as Ec=0.6, G=0 and γ=0.139; (b) suction parameter as G=0 and one surfactant value (λ)=1.5090; (c) surfactants parameter as Z=1.4881, Ec=0.6, G=0\cos wt and γ=0.139; (d) surfactants parameter as Ec=0.6, G=G_0 \cos wt and γ=0.139; (e) suction parameter as G=G_0 \cos wt and one surface tension (γ)=0.139.

Figure 3a. Figure 3d present the Temperature distributions of various values of surfactant (λ) with a value of surface tension (γ) = 0.129, when the pressure gradient is Harmonic in nature (G=G_0 \cos wt) at the interface of micro-droplets. It is clear that the Temperature profile (θ) of the droplets maintain its peak when the spatial direction (y) of the system is 0.60 with maximum and minimum temperatures of 0.9115272474 and 0.7178662138 respectively, the reduction in temperature was due to consistent increase in surfactant λ at the interface though the temperature here is more than Figure 3a but it is consistent with or without the concentration of surfactants. Figure 3e present the Temperature distributions of various values of suction parameter with a value of surface tension (γ) 0.129 and a surfactant (λ), when the pressure gradient is harmonic in nature (G=G_0 \cos wt) at the interface of micro-droplets. This shows that the Temperature profile (θ) of the droplets maintain its peak when the spatial direction (y) of the system is 0.60 with minimum and maximum temperatures of 0.002648568488 and 0.01231991189 respectively, the increase in temperature was due to less value of surfactant which will also increase the time of interfacial penetration.

Figure 4a present the Temperature distributions of various values of surfactant (λ) with a value of surface tension (γ) = 0.139, when the pressure gradient is zero (G=0) at the interface of micro-droplets. It is clear that the Temperature profile (θ) of the droplets maintain its peak when the spatial direction (y) of the system is 0.40 with maximum and minimum temperatures of 0.0000436927483 and 0.00002509847656 respectively, the reduction in temperature was due to consistent increase in surfactant λ at the interface. Figure 4b present the Temperature distributions of various values of suction parameter with a value of surface tension (γ) 0.139 and a surfactant (λ), when the pressure gradient is zero (G=0) at the interface of micro-droplets. This shows
that the Temperature profile ($\theta$) of the droplets maintain its peak when the spatial direction ($y$) of the system is 0.40 with minimum and maximum temperatures of 0.0002172278719 and 0.0001282548905 respectively, the increase in temperature was due to less value of surfactant which will also increase the time of interfacial penetration. Figure 4c also present the different combinations of surfactant values ($\lambda$) at the interface of micro droplets when the pressure gradient ($G = G_0 \cos \omega t$). It is clear seen that the Temperature profile ($\theta$) of the droplets maintain its peak when the spatial direction ($y$) of the system is 0.40 with maximum and minimum temperatures of 0.001852904916 and 0.001072304810 respectively, the reduction in temperature was due to consistent increase in surfactant $\lambda$ at the interface but not as low compare to Figure 4a. Figure 4d present the Temperature distributions of various values of surfactant ($\lambda$) with a value of surface tension ($\gamma$) = 0.139, when the pressure gradient is Harmonic in nature ($G = G_0 \cos \omega t$) at the interface of micro-droplets. It is clear that the Temperature profile ($\theta$) of the droplets maintain its peak when the spatial direction ($y$) of the system is 0.60 with maximum and minimum temperatures of 1.0087237674 and 0.6240424099 respectively, the reduction in temperature was due to consistent increase in surfactant $\lambda$ at the interface though the temperature here is more than Figure 4a but it is consistent with or without the concentration of surfactants. Figure 4e present the Temperature distributions of various values of suction parameter with a value of surface tension ($\gamma$) 0.139 and a surfactant ($\lambda$), when the pressure gradient is harmonic in nature ($G = G_0 \cos \omega t$) at the interface of micro-droplets. This shows that the Temperature profile ($\theta$) of the droplets maintain its peak when the spatial direction ($y$) of the system is 0.60 with minimum and maximum temperatures of 0.003076438360 and 0.01072304810 respectively, the increase in temperature was due to less value of surfactant which will also increase the time of interfacial penetration.

**DISCUSSION**

In this research work, the impact of surfactants on surface tension of microdroplets of unsteady incompressible flow considering velocity and temperature distribution at interface has been studied. The surfactants were assumed to vary with velocity and temperature with little increment, the suction parameters and viscous dissipation were taken into consideration.

The most interesting result is the fusion of different droplet with velocity coming to form a stable droplet with very low temperature and on the three levels of Pressure Gradients (Constant, Exponential and Harmonic). We must not forget that many of pharmaceutical processes depend on the cohesive and adhesive interactions between the materials used during the preparation of the products. Understanding and determination of surface free energies of both liquid and solid surfaces plays a key role in characterization of materials during their development, formulation and manufacturing of pharmaceutical applications. That is why surface chemistry has a large influence in many industries. The application of knowledge of surface tension is of utmost importance to yield new and better performing products. Surface tension can influence the development, production and performance of pharmaceutical, food, biomaterial and other products.

Consideration of parameters such as surfactant and surface tension of microdroplets may provide the critical component to solve industrial problems and improve product quality.

**Conclusion**

The above studies on surface tension of microdroplets lead to the following conclusion:

(a) The presence of surfactant at interface reduces the surface tension: chemical penetration enhancers such as surfactants, interact with keratin, swell stratum comeum and extract the intercellular lipid matrix of the stratum comeum.

(b) Reducing interfacial tension improves the stability of a droplet: which is in line with improving the contact between the drug and skin leads to enhanced permeation of the drug through stratum corneum.

(c) Laplace pressure gradient for water drops of different radii determine the stability of the drops: Therefore, the composition of a dung vehicle should be considered in its nomenclature. Some products have low surface tensions and spread rapidly and easily on the surface of skin, while others are difficult to apply to the surface.

(d) There is no life without surfactant: Respiratory Distress Syndrome (RDS) is common and potentially fatal disease in premature infants that is caused by insufficient surfactant in the lungs. Surfactant is a protein that reduces alveolar surface tension, enabling proper lung inflation and aeration. In most full-term infants, surfactant ensures soft and pliable lungs that stretch and contract with each breathe. Premature infants have underdeveloped lungs with insufficient surfactant and consequently their lungs are stiff and do not inflate easily.

(e) We also observed that combinations of surfactant values reduces the tension at the interface of homogenous droplets, also the velocity of all droplets are close because of the surfactants though, the velocity values are high compare to other pressure gradients but it satisfied the stability of the system.

(f) We also observed that low value of surfactant has no effect on higher value of surface tension at interface of homogenous fluids, harmonic value as pressure gradient obviously revealed the effect of surface tension at the
droplet interface with and without surfactant. (g) Harmonic values as pressure gradients stabilized this system more than Constant and Exponential values. We also observed that combinations of surfactant values reduces the tension at the interface of homogenous droplets, also the velocity of all droplets are close because of the surfactants though, the velocity values are high compare to other pressure gradients but it satisfied the stability of the system. (h) Harmonic values as pressure gradients stabilized this system more than Constant and Exponential values though all the velocity values are negative but it is more consistent on this system. (i) We also observed that low value of surfactant has no effect on higher value of surface tension at interface of homogenous fluids, harmonic value as pressure gradient obviously revealed the effect of surface tension at the droplet interface with and without surfactant. (j) We also observed that surfactant penetrate the interface with low temperature, the temperature of all droplets are close because of the concentration of surfactants. We also observed that droplet depends on pressure gradient.

REFERENCES


